

Evaluate the following integrals.

SCORE: ____ / 60 PTS

[a] $\int \frac{5+6x-3x^4}{x^3+2x^2+x} dx$

$$\begin{array}{r} x^3+2x^2+x \overline{) -3x+6} \\ \underline{-3x^4+0x^3+0x^2+6x+5} \\ -3x^4-6x^3-3x^2 \\ \underline{6x^3+3x^2+6x} \\ 6x^3+12x^2+6x \\ \underline{-9x^2} \quad +5 \end{array}$$

$$\int (-3x+6 + \frac{5-9x^2}{x(x+1)^2}) dx$$

$$= \int (-3x+6 + \frac{5}{x} + \frac{-14}{x+1} + \frac{4}{(x+1)^2}) dx$$

$$= \frac{-\frac{3}{2}x^2+6x}{2} + \frac{5 \ln|x| - 14 \ln|x+1| - \frac{4}{x+1} + C}{4} \quad \text{②} \quad \text{④} \quad \text{③} \quad \text{①}$$

$$\frac{5-9x^2}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \quad \text{④}$$

$$5-9x^2 = A(x+1)^2 + Bx(x+1) + Cx \quad \text{④}$$

$x=0: 5 = A$

$x=-1: -4 = -C \rightarrow C=4$

$x^2: -9 = A+B \rightarrow B = -9-A = -14$

SANITY CHECK:

$$\left. \begin{array}{l} x=-2 \\ \frac{5-12-48}{-8+8-2} \stackrel{?}{=} 6+6+\frac{5}{-2} - \frac{14}{-1} + \frac{4}{1} \\ \frac{55}{2} \stackrel{?}{=} 12 - \frac{5}{2} + 18 = 30 - \frac{5}{2} \\ = \frac{55}{2} \checkmark \end{array} \right\} \text{④}$$

[b] $\int \sec^2 \sqrt{x} dx$

④ $u = \sqrt{x} \rightarrow x = u^2$
 $dx = 2u du$

④ $\int 2u \sec^2 u du$

$\frac{u}{2u}$	$\frac{du}{\sec^2 u}$
$2u$	$\tan u$
2	$\ln \sec u $
0	

$= 2u \tan u - 2 \ln|\sec u| + C$ ④

$= 2\sqrt{x} \tan \sqrt{x} - 2 \ln|\sec \sqrt{x}| + C$ ③ ①

Evaluate the following integrals.

SCORE: ___ / 60 PTS

[a] $\int x^2 \sqrt{x^2 + 4x} dx = \int x^2 \sqrt{(x+2)^2 - 4} dx$

$(x+2)^2 - 4 = 4(\sec^2 \theta - 1) = 4 \tan^2 \theta$

$(x+2)^2 = 4 \sec^2 \theta$

$\sec \theta = \frac{x+2}{2}$

$x = 2 \sec \theta - 2$ (6)

$dx = 2 \sec \theta \tan \theta d\theta$

$\int (2 \sec \theta - 2)^2 (2 \tan \theta) (2 \sec \theta \tan \theta) d\theta$

$= 16 \int (\sec^2 \theta - 4 \sec \theta + 4) \sec \theta \tan^2 \theta d\theta$

$= 16 \left[\int \sec^3 \theta \tan^2 \theta - 4 \int \sec^2 \theta \tan^2 \theta d\theta + 4 \int \sec \theta \tan^2 \theta d\theta \right]$

$= 16 \int \sec^3 \theta (\sec^2 \theta - 1) d\theta$

$- 64 \left(\frac{1}{3} \tan^3 \theta \right)$

$+ 64 \int \sec \theta (\sec^2 \theta - 1) d\theta$

$= 16 \int \sec^5 \theta d\theta + 48 \int \sec^3 \theta d\theta - 64 \int \sec \theta d\theta - \frac{64}{3} \tan^3 \theta$

$= 16 \left[\frac{1}{4} \sec^3 \theta \tan \theta + \frac{3}{4} \int \sec^3 \theta d\theta \right] + 48 \int \sec^3 \theta d\theta - 64 \int \sec \theta d\theta$

$= 4 \sec^3 \theta \tan \theta + 60 \int \sec^3 \theta d\theta - 64 \int \sec \theta d\theta - \frac{64}{3} \tan^3 \theta$

$= 4 \sec^3 \theta \tan \theta + 30 \sec \theta \tan \theta + 30 \ln |\sec \theta + \tan \theta|$

$= 4 \sec^3 \theta \tan \theta + 30 \sec \theta \tan \theta - 34 \ln |\sec \theta + \tan \theta| - \frac{64}{3} \tan^3 \theta + C$

$= 4 \left(\frac{x+2}{2} \right)^3 \frac{\sqrt{x^2+4x}}{2} + 30 \left(\frac{x+2}{2} \right) \frac{\sqrt{x^2+4x}}{2} - 34 \ln \left| \frac{x+2}{2} + \frac{\sqrt{x^2+4x}}{2} \right| - \frac{64}{3} \left(\frac{\sqrt{x^2+4x}}{2} \right)^3 + C$

$= \frac{1}{4} (x+2)^3 \sqrt{x^2+4x} + \frac{15}{2} (x+2) \sqrt{x^2+4x} - 34 \ln |x+2 + \sqrt{x^2+4x}| - \frac{8}{3} (x^2+4x) \sqrt{x^2+4x}$

$+ C$

[b] $\int e^{-2x} \cos \frac{x}{2} dx = 2e^{-2x} \sin \frac{x}{2} - 8e^{-2x} \cos \frac{x}{2}$

$\frac{u}{e^{-2x}} + \cos \frac{x}{2}$
 $-2e^{-2x} - 2 \sin \frac{x}{2}$
 $4e^{-2x} - 4 \cos \frac{x}{2}$

$17 \int e^{-2x} \cos \frac{x}{2} dx$

$= 2e^{-2x} \sin \frac{x}{2} - 8e^{-2x} \cos \frac{x}{2}$

$\int e^{-2x} \cos \frac{x}{2} dx = \frac{2}{17} e^{-2x} \sin \frac{x}{2} - \frac{8}{17} e^{-2x} \cos \frac{x}{2} + C$

$u = \tan \theta$
 $du = \sec^2 \theta d\theta$
 $\int u^2 du$

Find $\int_0^{2\pi} \frac{\cos x}{1+\sin x} dx.$

$1 + \sin x = 0 \rightarrow \sin x = -1 @ x = \frac{3\pi}{2}$

SCORE: ___ / 30 PTS

(9) $\int_0^{\frac{3\pi}{2}} \frac{\cos x}{1+\sin x} dx + \int_{\frac{3\pi}{2}}^{2\pi} \frac{\cos x}{1+\sin x} dx$

$\int \frac{\cos x}{1+\sin x} dx$ $u = 1 + \sin x$
 $du = \cos x dx$
 $= \int \frac{1}{u} du$
 $= \ln|u|$
 $= \ln|1 + \sin x|$

(4) $\lim_{N \rightarrow \frac{3\pi}{2}^-} \ln|1 + \sin x| \Big|_0^N$ (5)

$= \lim_{N \rightarrow \frac{3\pi}{2}^-} \ln|1 + \sin N|$

(4) $= -\infty$

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INTEGRAL DIVERGES
 (4)